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Blockage correction with a free surface

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A simple analysis shows that with a disturbance present the potential jump in a steady flow in a canal is expressed in terms of (1) the effective volume (displaced volume and added mass/density of fluid) and (2) the depth Froude number for either a submerged body or a body with thin waterplane area. For a ship moving in a canal, the expression for potential jump contains a contribution from the line integral term along the intersection between the ship hull and the free surface. When a pressure distribution is given on the free surface, the potential jump can be expressed explicitly in terms of the depth Froude number and the total pressure force, regardless of the shape of the pressure distribution. From the present relations, the added mass of a ship in steady motion in a canal is computed from the potential jump computed previously by the author for various Froude numbers. This added mass plays an essential role in the equation of motion *initially* when a sudden external force is applied to a steady moving ship. The present analysis is complimentary to that of Newman (1976) and the extension of that to the three-dimensional case. As practical applications of the potential jump, which has had a limited interest, we proposed approximate formulas for speed correction and sinkage of a ship in a towing tank experiment. Also proposed is an approximate formula for the speed correction in a wind tunnel experiment. The present approximate formula is compared with 'exact' numerical results obtained by the localized finite element method for both towing tank and wind tunnel experiments. The present speed correction formula is also compared with existing approximate formulas for a wind tunnel experiment. The present formulas compare favourably with the exact numerical results.

1. Introduction

The occurrence of blockage, or a jump in the velocity potential between upstream and downstream infinities, is well known for steady flow past a disturbance in a canal having a finite cross-sectional area. In his analysis of the two-dimensional case with a free surface, Newman (1976) shows that the potential jump may be expressed in terms of the doublet strength and the depth Froude number. In this paper we describe a different and simpler analysis, from which an expression for the potential jump for a general, three-dimensional disturbance is obtained; when the disturbance is due to a ship, the potential jump is expressed in terms of the effective volume (displaced volume and added mass/density), the depth Froude number, and a line integral term along the interface between the free surface and the ship hull. When the body is fully submerged, the line integral term disappears, and the potential jump is expressed only in terms of the depth Froude number and the effective volume. From the

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classical relation for an infinite fluid, by which the doublet strength is just the effective volume, the present analysis not only confirms the earlier result of Newman but also validates the classical relation when a free surface is present. The expression obtained is also valid as gravitational acceleration tends to infinity, i.e. the free surface becomes rigid. Previously the potential jump in a free-surface flow was considered to be of limited physical interest, except for its practical importance in the numerical solution of canal problems first noted by Bai (1975) and Chen & Mei (1975). Potential jumps were also discussed by Bai (1978*a*) for two-dimensional hydrofoil problems and in Bai (1977) for three-dimensional problems.

Blockage effects in wind tunnel tests have been recognized for a long time, and it has been routine practice to make theoretical blockage corrections to wind tunnel experimental data. Ship hydrodynamicists have subsequently investigated towing-tank blockage effects due to tank sidewalls and a finite depth bottom.

Owing to the difficulty encountered in computing flow separation, wake flow, free-surface effects, etc., the exact magnitude of the blockage effect on fluid force acting on a body is too complicated to analyse by purely theoretical means. However, these difficulties did not stop engineers from attempting to make simple engineering approximations of the blockage problem. For engineering purposes, computation of a mean-speed increment on a body owing to blockage effects has been the main focus of interest so as to make blockage correction to frictional drag. In the computation, the incremented change in frictional drag owing to blockage is determined directly from the computed incremental increase of mean speed over the body surface caused by flow blockage.

Two basic inviscid-flow theories have been previously employed. The first is based on the so-called, one-dimensional mean-flow theory, the Kreitner equation, which was first obtained by Kreitner (1934) using from Bernoulli and mass continuity equations under the assumption that velocity is uniform in each cross-sectional plane. To name a few, Hughes (1961) and Kim (1963) used this approach.

The second, based on a successive reflexion of images in the walls of the rectangular tank or simpler axisymmetric singularities in case of axisymmetric flows. In this approach, the velocity potential of the flow inside a specified tank boundary can be computed exactly in principle; usually, the potential is represented by a series expansion, and only the first few terms are computed. Ogiwara (1975), Tamura (1972, 1975) and Landweber & Nakayama (1975) have used the latter approach.

In all, there exist about a dozen formulas proposed for blockage corrections, and each is somewhat different from the other. Some formulas introduce empirical correction factors, whereas others claim to be based on analytical derivations. Some formulas are proposed to be used only for frictional resistance corrections, whereas other formulas are used for total resistance corrections. An extensive review of the subject has been made by Gross & Watanabe (1972).

The main objectives of the present paper are twofold: (1) to obtain an expression for the potential jump in terms of the depth Froude number and the effective volume for free-surface flows in general, and (2) to obtain an approximate speed-correction formula and test the present approximate formulas. A comparison is made between the results obtained by the present approximate formulas and those obtained by exact numerical results. Numerical results are obtained by a localized finite element method developed by Bai (1977). As a test of the present speed-correction formula, two cases are considered: (*a*) the Wigley parabolic ship model, tested in both a small and a large towing tank, (*b*) a body of revolution (prolate spheroid) tested in a circular wind tunnel. In each, the mean-speed increment averaged over the entire body surface is computed by a three-dimensional, finite element method applicable to free-surface flow problems. These compare favourably with those obtained by the approximate speed-correction formula. Results are also compared to those obtained by using the speed-correction formula of Lock and Johansen. The present formula renders a better approximation than that of Lock and Johansen when the cross-sectional areas of a flow tunnel is not much larger than the maximum cross-sectional area of the body.

We will first consider steady, free-surface flow in three-dimensions to obtain an expression for the potential jump. The potential jump in the special case when the disturbance is specified on the canal boundary (and on the free surface) is given explicitly in terms of a non-homogeneous boundary condition. In § 3, as applications of the expression for the potential jump obtained in the present analysis, approximate formulas for the mean-speed correction on the body are proposed and compared with the 'exact' mean-speed correction for both towing tank and wind tunnel experimental conditions. Also given is an approximate expression for additional sinkage of a ship which is due to the blockage effect in confined waterways.

2. Mathematical analysis

We consider here steady uniform flow past a fixed three-dimensional disturbance in a canal with rectangular, uniform cross-section. The co-ordinate system is right-handed and rectangular. The y axis is opposite to the force of gravity, and the x, z plane coincides with the undisturbed free surface. The bottom of the canal is in the $y = -H$ plane; the side walls, in the $z = \pm b$ planes. The uniform flow comes from the negative x axis on the left-hand side. Surface tension is neglected and it is assumed that the fluid is inviscid and incompressible, and the motions are irrotational. In the following analysis, the two-dimensional case becomes a special case by taking the tank width ($2b$) as unit length and by assuming the disturbance is the same on all planes perpendicular to the z axis.

The steady three-dimensional flow in a canal is described by a total velocity potential

$$\Phi(x,y,z) = U(x+\phi(x,y,z)), \tag{2.1}$$

where ϕ is the perturbation potential normalized with respect to the uniform stream U. Then

$$\nabla^2\Phi = \nabla^2\phi = 0 \tag{2.2}$$

in the fluid domain D.

It will be assumed that the free-surface disturbances are all small so that the linearized free-surface boundary condition,

$$U^2\phi_{xx} + g\phi_y = \begin{Bmatrix} 0 & \text{on} \quad S_F \\ -\dfrac{1}{\rho}\dfrac{\partial}{\partial x}P_0(x,z) & \text{on} \quad S_P \end{Bmatrix}, \tag{2.3}$$

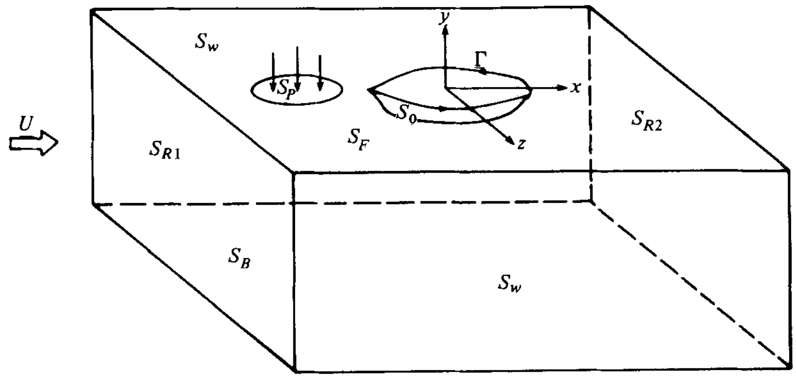

FIGURE 1. Boundary configurations of the fluid domain D.

can be applied, where $p_0(x, z)$ is a specified non-zero pressure distribution applied over a portion of the free surface S_P, S_F is the free surface excluding S_P, and ρ is the density of water. The boundary condition on the ship hull S_0 is

$$\phi_n = -n_1, \tag{2.4}$$

where $\mathbf{n} = (n_1, n_2, n_3)$ is the unit normal vector outwards from the fluid. The boundary conditions on the bottom S_B and side walls S_W of a canal are

$$\phi_n = 0 \quad \text{on} \quad S_B \cup S_W. \tag{2.5}$$

As the radiation condition, we require that the potential behaves as

$$\lim_{x \to -\infty} \phi = 0 \tag{2.6}$$

on a plane S_{R1} at the upstream infinity, i.e. $x \to -\infty$, and

$$\lim_{x \to +\infty} \phi = K + \overline{\phi}(x, y, z) \tag{2.7}$$

on a plane S_{R2} at the downstream infinity, where the constant K is the potential jump, and $\overline{\phi}$ represents the free-wave term. Here $\overline{\phi}$ can be expressed as

$$\overline{\phi} = \sum_{n=0}^{\infty} (a_n \cos k_n x + b_n \sin k_n x) \frac{\cosh \mu_n (y+H)}{\cosh \mu_n H} \cos \frac{n\pi}{2b} (z-b), \tag{2.8}$$

where a_n and b_n are coefficients yet to be determined, and μ_n and k_n are the positive roots of

$$\frac{U^2}{gH} \left[\mu H - \left(\frac{n\pi H}{2b} \right)^2 \frac{1}{\mu H} \right] = \tanh \mu H \tag{2.9}$$

and

$$k_n^2 = \mu_n^2 - \left(\frac{n\pi}{2b} \right)^2.$$

If $n \geq 1$, there is a single solution for μ_n for each value of U^2/gH. Where $n = 0$, there is one positive solution for μ_0 if $U^2/gH < 1$, but none if $U^2/gH > 1$.

Let us apply Green's theorem to the functions x and ϕ, defined in the fluid domain D and its boundary ∂D as shown in figure 1. We obtain

$$\iint_{\partial D} \left(\phi \frac{\partial}{\partial n} x - x \frac{\partial}{\partial n} \phi \right) ds = 0, \tag{2.10}$$

where $$\partial D = S_F \cup S_P \cup S_0 \cup S_W \cup S_B \cup S_{R1} \cup S_{R2}.$$

Note that the relation (2.10) has been used also in Birkhoff (1950), Landweber (1956), Landweber & Yih (1956) and Yeung (1977). By substituting the proper boundary conditions for ϕ given in (2.3) through (2.6).

$$\iint_{S_0}\left(\phi\frac{\partial}{\partial n}x - x\frac{\partial}{\partial n}\phi\right)ds + \iint_{S_{R2}}(\phi - x\phi_x)\,ds + \frac{1}{\rho g}\iint_{S_p} x\frac{\partial}{\partial x}P_0\,ds + \frac{U^2}{g}\iint_{S_F\cup S_p} x\phi_{xx}\,ds = 0. \quad (2.11)$$

Integrating by parts, and using (2.7), (2.11) reduces to

$$\iint_{S_0}(\phi n_1 - \phi_n x)\,ds - \frac{1}{\rho g}\iint_{S_p} P_0\,ds + \frac{U^2}{g}\oint_{\Gamma}(x\phi_x - \phi)\,dz + \lim_{x\to\infty}\int_{-b}^{b} dz\left[\frac{U^2}{g}\{x\overline{\phi}_x(x,0,z) - \overline{\phi}(x,0,z)\} - \int_{-H}^{0}\{x\overline{\phi}_x(x,y,z) - \overline{\phi}(x,y,z)\}\,dy\right] + KWH\left(1 - \frac{U^2}{gH}\right) = 0, \quad (2.12)$$

where Γ is the intersection line between the free surface S_F and the ship hull surface S_0, and where $W = 2b$ is the width of the canal. Here the line integral along the closed contour Γ is understood to proceed in the counter-clockwise direction when we look down at the free surface, i.e. $dz < 0$ around the stern and $dz > 0$ around the bow. In (2.12), the line integral along the boundary contour Γ_P of S_p, $\oint_{\Gamma_P} xP_0\,dz$ is eliminated by assuming $P_0 = 0$ on Γ_P. If desired this line integral along Γ_P may be included in the present analysis in a straightforward manner. Here x will be assumed to be finite at first and later will be taken in the $\lim x \to \infty$. By using the orthogonality relations for $\overline{\phi}$,

$$\int_{-b}^{b}\left[\int_{-H}^{0}\overline{\phi}\,dy - \frac{U^2}{g}\overline{\phi}(x,0,z)\right]dz = 0 \quad (2.13a)$$

and
$$\int_{-b}^{b}\left[\int_{-H}^{0}\overline{\phi}_x\,dy - \frac{U^2}{g}\overline{\phi}_x(x,0,z)\right]dz = 0. \quad (2.13b)$$

Equation (2.12) further reduces to

$$KWH(1 - F_H^2) = \iint_{S_0}(x\phi_n - \phi n_1)\,ds + \frac{1}{\rho g}\iint_{SP} P_0\,ds + \frac{U^2}{g}\oint_{\Gamma}(\phi - x\phi_x)\,dz, \quad (2.14)$$

where $F_H = U/\sqrt{gH}$ is the depth Froude number. (Note in (2.13) that the z integrals vanish for $n \neq 0$ and the y integrals vanish for $n = 0$.) Equation (2.14) may be rewritten in a form

$$KWH(1 - F_H^2) = V(1 + m') + \frac{F_0}{\rho g} + \frac{U^2}{g}\oint_{\Gamma}(\phi - x\phi_x)\,dz, \quad (2.15)$$

where V is the submerged volume of the ship hull under the free surface, the 'added mass' m and its coefficient m' are defined, respectively, by

$$m = -\rho\iint_{S_0}\phi n_1\,ds \quad (2.16a)$$

and
$$m' = m/\rho V,$$

and F_0 is the total pressure force applied on the free surface. The line integral in equation (2.15) is similar to the line integral which is present in Green's function formulation of the wave resistance problem. The contribution of the line integral to the potential jump requires careful analysis. If we restrict ourselves to thin ship hulls, we may assume the line integral contribution in (2.15) to be negligibly small. (Some discussion on the line integral has been given in Bai, 1977.)

It is worthwhile to mention that the added mass defined by (2.16*a*) in the steady wave resistance formulation can be interpreted for two problems. First, when an external force $F\bar{H}(t)$ is suddenly applied to a ship moving with a constant velocity (where F is a constant, and $\bar{H}(t)$ is the Heaviside function) the acceleration (or deceleration depending on the sign of F) of the ship is given by $F/\rho V(1+m')$ at $t = +0$.† Second, the added mass defined in equation (2.16) can be interpreted as the surge added mass for the limiting case of zero-frequency harmonic surge motion of a body in an otherwise steady uniform stream. (Specifically, when $\sigma l/U$ approaches zero, where σ is the frequency of surge motion, and l is an appropriate length scale.) Here, the boundary condition in the body is applied to the mean position of the body boundary as in the customary linear formulation.

The result can be simplified for two-dimensional problems, by taking the canal width $W = 1$, to

$$KH(1-F_H^2) = S(1+m') + \frac{F_0}{\rho g} + \frac{U^2}{g}[\phi - \phi_x x]_{x_2}^{x_1}, \tag{2.17}$$

where S is the submerged cross-sectional area, of the two-dimensional body and the contour integral along Γ in equation (2.15) is reduced to two point values, if the body pierces the free surface. Here the intersection points x_1 and x_2 are the x co-ordinates of the free surface at the upstream and downstream ends of the body, respectively. Investigation of a surface-piercing body in two dimensions is beyond the scope of the present work since linearization of the free surface totally breaks down near the intersection points.

If the body is submerged with no pressure distribution on the free surface, and if g tends to infinity, then equations (2.15) and (2.17) reduce to

$$K = (1+m')\,V/A \tag{2.18}$$

in three dimensions, where $A = WH$ is the cross-sectional area of the canal, and

$$K = (1+m')\,S/H \tag{2.19}$$

in two dimensions. Here $(1+m')\,V$ and $(1+m')\,S$ are the so-called effective volume and the effective area, respectively. Equation (2.19) has been discussed in Newman [1969, equation (3.4)] for the free surface becoming rigid. We will discuss (2.18) further when we consider the wind-tunnel blockage correction later.

When the body is fully submerged or when the line integral can be ignored for a thin, surface-piercing ship without an imposed pressure distribution on the free surface, (2.15) and (2.17) can be reduced, respectively, to

$$KA(1-F_H^2) = (1+m')\,V \tag{2.20}$$

† Strictly speaking, a more complicated convolution intergal has to be computed for any $t > 0$; this problem has to be treated as an initial value problem such as considered by Wehausen (1964) and Calisal (1977).

in three dimensions and

$$KH(1-F_H^2) = (1+m')S \tag{2.21}$$

in two dimensions. In his analysis in two dimensions, Newman (1976) obtained the following expression for the potential jump.

$$KH(1-F_H^2) = 2\pi\mu, \tag{2.22}$$

where μ is the doublet strength. By a straightforward extension of Newman's analysis to the three-dimensional case, we obtain

$$KA(1-F_H^2) = 4\pi\mu \tag{2.23}$$

in three dimensions. From equations (2.20) and (2.23), for three dimensions

$$\mu = \frac{1+m'}{4\pi}V. \tag{2.24}$$

Similarly, from (2.21) and (2.22)

$$\mu = \frac{1+m'}{2\pi}S \tag{2.25}$$

in two dimensions.

The present analysis, which is simple and complimentary to Newman's analysis, provides some physical understanding of the role of the added mass in the potential jump. It is of interest to note the relations (2.24) and (2.25) are the classical results in an infinite fluid. The relation (2.24) was discussed by Taylor (1928). More general cases of (2.24) in an infinite fluid without a free surface were discussed by Cummins (1953), Landweber (1956) and Landweber & Yih (1956). A neat derivation of (2.24) and (2.25) in an infinite fluid can also be found in Newman (1977, p. 143). From the present analysis it is shown that the classical relations in (2.24) and (2.25) are also valid when a free surface is present.

When the hull boundary condition is applied on the centre-plane of the ship, which is the customary linearized hull boundary condition under the conventional thin-ship approximation, i.e.

$$\phi_n(x,y,\pm 0) = \pm\frac{\partial}{\partial x}f(x,y), \tag{2.26}$$

where the ship hull boundary is given by

$$z \pm f(x,y) = 0. \tag{2.27}$$

Then the integral on the ship surface S_0 in (2.14) reduces to

$$\begin{aligned}\iint_{S_0}(x\phi_n-\phi n_1)\,ds &= 2\iint_{C_p+} x\frac{\partial}{\partial x}f(x,y)\,dx\,dy - 2\iint_{C_p+}\phi n_1\,dx\,dy \\ &= V,\end{aligned} \tag{2.28}$$

where the surface of integration C_{P+} is the positive side of the centre-plane of the ship ($z = +0$), and the added mass term defined by the second integral on the right-hand side in equation (2.28) become trivially zero. In equation (2.28) the first term in the integral on the left-hand side reduces to the displaced volume when either the exact hull condition or the linearized hull condition as shown previously is used. However, the second term in the same integral differs since this term becomes zero

Cases	(a) Three dimensions	(b) Two dimension
1	$\dfrac{V(1+m')}{A(1-F_H^2)}$	$\dfrac{S(1+m')}{H(1-F_H^2)}$
2	$\dfrac{V}{A(1-F_H^2)}$	$\dfrac{S}{H(1-F_H^2)}$
3	$\dfrac{V(1+m')}{A}$	$\dfrac{S(1+m')}{H}$
4	$\dfrac{V}{A}$	$\dfrac{S}{H}$
5	$\dfrac{F_0}{\rho g A(1-F_H^2)}$	$\dfrac{F_0}{\rho g H(1-F_H^2)}$

Note. 1. Line integral is negligibly small.
2. 1 and m' are small, or body boundary condition is linearized.
3. Gravity g approaches to infinity, e.g. wind-tunnel test.
4. 3 and m' are negligibly small, or body boundary condition is linearized.
5. Pressure distribution on free surface is given.

TABLE 1. A summary of expressions for potential jump K.

for the linearized hull condition. From equations (2.14) and (2.28), we obtain for the linearized hull condition.

$$K(1-F_H^2) = V/A. \tag{2.29}$$

Here, no pressure distribution is present, and the ship is assumed to move along the centre-line of the canal.

An improved approximation of the added mass defined in (2.16) may be computed by invoking the original hull boundary

$$m = -2\rho \iint_{C_p+} \phi(x,y,+0)\frac{\partial}{\partial x} f(x,y)\,dx\,dy. \tag{2.30}$$

The added mass given by equation (2.30) for a thin ship can be interpreted as a higher order approximation than the zero added mass given by (2.28).

The added mass in the expression for K in the present analysis is also trivially zero for a pressure distribution on the free surface. Notice that the value of $K(1-F_H^2)$ remains constant for all Froude numbers; this was noticed in earlier numerical solutions for both two- and three-dimensional problems in Bai (1975, 1977, 1978*a*). If only a pressure distribution is present, (2.15) reduces to

$$K(1-F_H^2) = F_0/\rho g A. \tag{2.31}$$

Thus K depends only on the total pressure force and not on the shape of the distribution. Table 1 gives several cases and reduces from the general expressions (2.15) for three dimensions and (2.17) for two dimensions.

The relation in (2.31) has been also observed in previous numerical solutions made by Bai (1975, 1977, 1978*a*). In table 2 the values of $K(1-F_H^2)$ determined from (2.29) and (2.31) are compared with the numerical solutions for three dimensions given by Bai (1977). Here $BL/WH = 1$, and B and L are, respectively, the breadth and length of a rectangular pressure distribution with a constant value of P_0. For a thin ship, B and T are, respectively, the beam and draft of the ship, where $B/L = 0{\cdot}2$, $T/L = 0{\cdot}1$,

	Numerical results Bai (1977)	Present formula (2.29) or (2.31)
Rectangular pressure distribution	$P_0/\rho g H$	$F_0/\rho g A H = P_0/\rho g H$
Thin ship	0·14815	$\dfrac{V}{AH} = \dfrac{4}{27} = 0{\cdot}148\ldots$

TABLE 2. Comparisons of potential jump $K(1-F_H^2)/H$.

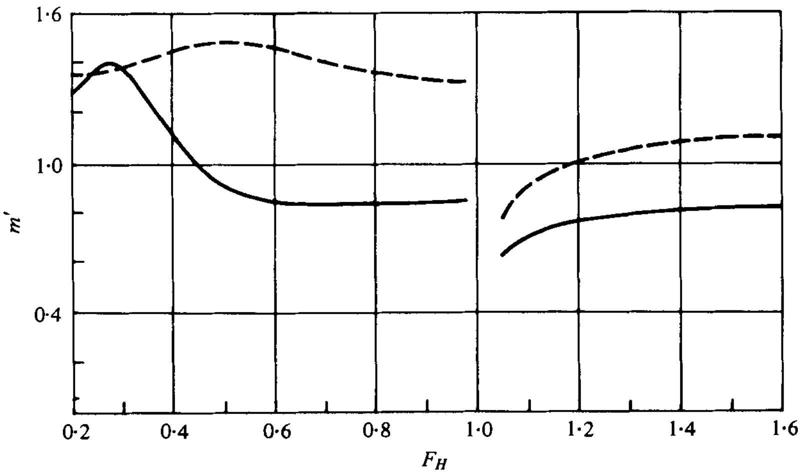

FIGURE 2. Added mass coefficients, m', computed from (2.19).
——, circle, $H/h = 5, h/a = 2, S = \pi a^2$; – – –, ellipse at $\pm 30°, a/b = 4, H/h = 2, H/a = 4, S = \pi ab$.

$H/L = 0{\cdot}3$, $W/L = 1$, and the ship has a vertical wallsided parabolic hull. Table 2 shows excellent agreement for both a pressure distribution and a thin-ship approximation.

By using the present results the added mass can easily be computed from the potential jump given by Bai (1975, 1978*a*) for two-dimensional cases and by Bai (1977) for three-dimensional cases. The added masses of a submerged circular cylinder in water of finite depth and an elliptical cylinder at a 30-degree angle of attack with zero circulation are given in figure 2. (A proof that the potential jumps are the same for a given body in either forward or reverse flow is given in Bai (1978*a*).) In figure 2, h is the depth of submergence, i.e. the distance from the free surface to the centre of a circular cylinder of radius a or of an elliptic cylinder of major and minor radii, a and b, respectively. The angle of attack is measured from the negative x axis to the major radius. Figure 2 shows that the added mass of a submerged circular cylinder reaches a maximum at approximately $F_H = 0{\cdot}275$, whereas the added mass of an elliptic cylinder submerged in water of finite depth with 30-degree angle of attack reaches its maximum approximately at $F_H = 0{\cdot}5$. The added mass coefficients for both bodies approach constant values asymptotically as the depth Froude number increases to $F_H = 1$. Table 3 gives the computed added mass coefficients of a vertical-sided parabolic ship where the line integral is ignored in the computations. Note that the added mass coefficient increases significantly as the length Froude number F_L [$= U/(gL)^{\frac{1}{2}}$] decreases.

F_L	m'
0·53	0·0630
0·50	0·0671
0·475	0·0750
0·45	0·0950
0·425	0·1268

TABLE 3. Added mass coefficient m' of a parabolic ship for various Froude number computed by formula (2.20) (Ship geometry is given in Bai, 1977).

3. Applications of potential jump

Let us consider the potential jump defined in the previous section. From the definition of the velocity potential, it can be written as

$$\phi = \int_{p_0}^{p} \frac{\partial}{\partial s}\phi \, ds + [\phi]_{p_0}, \tag{3.1}$$

where $\partial\phi/\partial s$ is the tangential velocity. When we integrate along a line parallel to the x axis,

$$\phi = \int_{-\infty}^{x} u \, dx, \tag{3.2}$$

where

$$\nabla\phi = (u, v, w).$$

Then the potential jump can be expressed as

$$K = \left[\int_{-\infty}^{\infty} u \, dx\right]_{\text{mean}}, \tag{3.3}$$

where by 'mean' under the bracket, it is understood that the free-wave components are to be precipitated. The potential jump can also be interpreted as the integral of the velocity increment owing to the blockage.

Blockage correction

Many authors have proposed approximate formulas for blockage corrections due to sidewalls and bottom boundary effects of a towing tanks. The first attacks on this subject date back more than four decades. Two basic approaches have been employed, one based on wind-tunnel correction; another, on a so-called mean-flow theory. There exist about a dozen formulas proposed for the blockage corrections; each different. Some formulas introduce empirical correction factors, and others claim to be based on analytical derivations. Some formulas are proposed to be used only for frictional resistance corrections, and others are used for total resistance corrections. An extensive review of this subject has been made by Gross & Watanabe (1972).

Here, we propose a new speed-correction formula to be used when analysing the frictional resistance of the ship model in a towing tank. A major difference between the present derivation and past derivations of speed-correction formulas is that in the present analysis, the potential jump occurring in a three-dimensional formulation

Cases	(a) Three dimensions	(b) Two dimensions
1	$\dfrac{V(1+m')}{LA(1-F_H^2)}$	$\dfrac{S(1+m')}{LH(1-F_H^2)}$
2	$\dfrac{V}{LA(1-F_H^2)}$	$\dfrac{S}{LH(1-F_H^2)}$
3	$\dfrac{V(1+m')}{LA}$	$\dfrac{S(1+m')}{LH}$
4	$\dfrac{V}{LA}$	$\dfrac{S}{LH}$
5	$\dfrac{F_0}{\rho g LA(1-F_H^2)}$	$\dfrac{F_0}{\rho g LH(1-F_H^2)}$

(See note in table 1)

TABLE 4. A summary of expressions for the mean-speed correction $\Delta\bar{u}$.

is used, whereas in the past, a one-dimensional, mean-flow theory or successive reflexion of images was used.

It is not easy to examine the local contribution to the potential jump, particularly when free waves are present. The potential jump K is the total speed increment in the x direction. As well as two-dimensional slender (or thin) bodies given in Bai (1975, 1977, 1978*a*), numerical solutions for practical ship forms and pressure distributions in three dimensions however, do indicate that major portion of the potential jump occurs along the hull length, especially for subcritical flow.

This evidence is also found in the numerical results of a potential flow model for the wind tunnel experiments, which will be shown later. It is also shown analytically that the major portion of K is confined only along the body length in a simple analysis for wind tunnel flows; see in appendix A. However, similarly, the same evidence can also be shown analytically for towing tank flows. This fact, proved analytically for a simple model in appendix A as well as observed in the numerical results, will be used as the basis for obtaining the following approximate mean-speed correction (or increment due to blockage effects)

$$\Delta\bar{u} = K/L, \tag{3.4}$$

where L is the ship length. In general, L need not be the body length but can be some characteristics length; for example, for a disk vertical to the stream, the body length is zero and therefore not an appropriate length. The previously defined speed-increment formula can be used for either two- or three-dimensional cases with or without a free surface. By substituting the expressions for K in table 1 into (3.4), we obtain table 4 for a mean-speed-correction formula.

Compare the present formula with blockage correction formulas obtained previously by using a simple mean-flow theory for a model towed in a tank. In this analysis, Hughes (1961) showed that an approximate formula for the speed correction was given by

$$\Delta\bar{u} = \frac{V}{LA(1-F_H^2-V/LA)}. \tag{3.5}$$

A similar formula which incorporated an empirical factor was also suggested by Kim (1963):

$$\Delta\bar{u} = \alpha \frac{V}{LA(1-F_H^2 - V/LA)}, \tag{3.6}$$

where α was obtained from experimental data.

To compare the present to the previous results, (3.5) may be written in a slightly different form, making use of the assumption that $V/LA(1-F_H^2)$ is small

$$\Delta\bar{u} = \frac{V}{LA(1-F_H^2)}\left(1+\frac{V}{LA(1-F_H^2)}\right)+o(1). \tag{3.7}$$

In his shallow water theory, Tuck (1967) obtained the local speed increment $\Delta u(x)$ for small values of W as

$$\Delta u(x) = \frac{S(x)}{A(1-F_H^2)}, \tag{3.8}$$

where $S(x)$ is the cross-sectional area of the ship. Tuck also showed that his result agrees with the results of a crude hydraulic theory, also given in his paper. The mean-speed-correction formula obtained from the previously described crude hydraulic theory by integrating (3.8) along the ship length may be given as

$$\Delta\bar{u} = \frac{1}{L}\int_{-\frac{1}{2}L}^{\frac{1}{2}L} \Delta u(x)\,dx = \frac{V}{LA(1-F_H^2)}. \tag{3.9}$$

If the gravity goes to infinity, i.e. the case of a wind tunnel test, (3.9) further reduces to a crude mean flow theory as

$$\Delta\bar{u} = V/LA. \tag{3.10}$$

It should be noted that (3.9) and (3.10) are identical to the cases 2(*a*) and 4(*a*) in table 4, where the added mass is assumed to be negligibly small.

Two specific cases will now be considered as tests of mean-speed-correction formula: (1) the Wigley parabolic ship model tested in both a small and large towing tank, using the three-dimensional formula of 2(*a*) table 4; (2) a body of revolution, i.e. prolate spheroid, tested in a circular wind tunnel with the three-dimensional formula of 3(*a*) in table 4. For comparison, the 'exact' mean-speed correction will also be computed from the numerical results obtained by the localized finite-element method developed by Bai (1977). We shall describe the 'exact' mean-speed correction as follows.

Exact mean-speed increment

To test the new mean-speed-correction formula proposed in the previous section, it will be necessary to consider the 'exact' mean-speed increment averaged over the entire body surface. Let the total velocity potential Φ_0 describe the same free-surface flows about the same body (or any sort of disturbance) described in § 2 but in an infinite half space below the free surface, i.e. in the absence of the canal boundaries. Then we have, similar to (2.1),

$$\Phi_0(x,y,z) = U(x+\phi_0(x,y,z)], \tag{3.11}$$

where ϕ_0 is the perturbation velocity potential in an unbounded fluid, normalized with respect to the uniform incoming stream U.

The fluid speed on a body surface, in general, increases due to the blockage effect when compared with that of unbounded fluid. However, the speed increment on the body surface is not uniform over the entire surface. For example, the forwarded stagnation point of an axisymmetric body remains the same whether in an unbounded fluid or in a wind tunnel of circular cross-section. Nevertheless, a mean-speed correction has been traditionally employed for the blockage correction, mainly owing to its simplicity. To describe a mean-speed increment due to blockage locally on the body surface

$$\Delta u = \nabla(\Phi - \Phi_0) . \boldsymbol{\tau} = \nabla(\phi - \phi_0) . \boldsymbol{\tau}, \tag{3.12}$$

where $\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$ is a unit tangential vector on the body surface; τ_1 is the component along the x axis, i.e. the longitudinal direction, and τ_2 and τ_3 are, respectively, the normal and tangential components in cross-sectional plane of the body. Then the 'exact' mean-speed increment averaged over the entire submerged body surface is given by

$$\Delta \overline{u} = \frac{1}{S_0} \iint_{S_0} \nabla(\phi - \phi_0) . \boldsymbol{\tau} \, ds. \tag{3.13}$$

Where S_0 is the wetted surface area, and $\boldsymbol{\tau}$ is specified. One natural way of specifying $\boldsymbol{\tau}$ would be as the unit potential-flow streamline vector on the body. However, streamlines on a body in bounded and unbounded flows, described by ϕ and ϕ_0, respectively, do not coincide in general, except in the special case of an axisymmetric body in a flow facility of circular cross-section. For a ship hull, if $\boldsymbol{\tau} = (1, 0, 0)$, and $S_0 = 2L . T$ under the assumption that the ship is sufficiently thin, (3.13) can be reduced to

$$\Delta \overline{u} = \overline{u} - \overline{u}_0,$$

where

$$\left.\begin{aligned} \overline{u} &= \frac{1}{L . T} \int_{-T}^{0} [\phi(\tfrac{1}{2}L, y, 0) - \phi(-\tfrac{1}{2}L, y, 0)] \, dy, \\ \overline{u}_0 &= \frac{1}{L . T} \int_{-T}^{0} [\phi_0(\tfrac{1}{2}L, y, 0) - \phi_0(-\tfrac{1}{2}L, y, 0)] \, dy, \end{aligned}\right\} \tag{3.14}$$

where L and T are the ship length and draft, respectively. Here the draft T is assumed to be uniform from the bow at $x = -\frac{1}{2}L$ to the stern at $x = \frac{1}{2}L$; the centre-plane of the ship is on $z = 0$.

Similarly, for a slender axisymmetric body of revolution in a wind tunnel of circular cross-section, the mean-speed increment averaged over the body surface is given by

$$\Delta \overline{u} = \overline{u} - \overline{u}_0$$

where

$$\left.\begin{aligned} \overline{u} &= \frac{1}{L} [\phi]_{x=-\frac{1}{2}L,\ R=0}^{x=\frac{1}{2}L,\ R=0}, \\ \overline{u}_0 &= \frac{1}{L} [\phi_0]_{x=-\frac{1}{2}L,\ R=0}^{x=\frac{1}{2}L,\ R=0}, \end{aligned}\right\} \tag{3.15}$$

where

$$R = (y^2 + z^2)^{\frac{1}{2}},$$

and the peripheral length along a body meridian is approximated by the body length, assuming that the body is slender.

In the following, to test of the present mean-speed-correction formula, two cases are specifically considered: (1) the Wigley parabolic ship model, tested in both a small and a large towing tank; (2) a body of revolution (prolate spheroid) tested in a

	Small tank	Large tank	Extra large tank
Width (m)	6·09	12·5	24
Mean water depth (m)	3·555	6·268	12

TABLE 5. Dimensions of small, large and extra large towing tanks.

Tank size	Exact numerical results from (3.14) $\bar{u}_0$	$\bar{u}$	$\Delta\bar{u} = \bar{u}-\bar{u}_0$	Formula 2(*a*) of table 4 $\Delta\bar{u}$
Small	0·017198	0·030514	0·0133	0·0128
Large	0·017198	0·019425	0·0022	0·0029

TABLE 6. Comparisons of mean-speed increment, computed from numerical results and the present formula for $F_L = 0{\cdot}4$.*

* Results of extra large tank were used to compute $\bar{u}_0$ as discussed in text.

circular wind tunnel. In each case the mean-speed increment averaged over the entire body surface is computed by a three-dimensional, finite element method applicable to free-surface flow problems. These are shown to compare favourably with those obtained by the approximate speed-correction formula. Results are also compared to those obtained by using the speed-correction formula of Lock and Johansen. The present formula renders a better approximation than that of Lock and Johansen when the cross-sectional area of a flow tunnel is not much larger than the maximum cross-sectional area of the body.

Towing tank experiment

To test the new blockage correction formula, three sets of computations were first made for the same ship in three different towing tanks. The first two tanks had the dimensions given by Tamura (1972, 1975); see table 5. The third tank was approximately four times greater in cross-sectional area than the large tank, i.e. $W = 24$ m and $H = 12$ m. The specific ship model considered was the Wigley parabolic model (model M1719 in Tamura) and the equation of the hull surface was given by where $L/B = 10$, and $T/L = 0{\cdot}0625$, and $L = 8$ m,

$$z = \pm\tfrac{1}{2}B\left\{1-\left(\frac{x}{\frac{1}{2}L}\right)^2\right\}\left\{1-\left(\frac{y}{T}\right)^2\right\}. \tag{3.16}$$

In the computations, the ship hull boundary condition was linearized; thus, the three-dimensional speed-correction formula (case 2(*a*) in table 4) was used. To test the present mean-speed-correction formula, computations were also made from (3.14); the exact mean-speed correction was averaged over the ship hull surface specifically, the centre-plane in this example) from the local velocities obtained by a localized finite element method of Bai (1977). In computing the values of $\bar{u}_0$ from (3.14), the numerical result for the extra large tank was used in place of the perturbation potential for unbounded water, because the effect of the tank walls and bottom boundaries was found to be negligibly small through numerical experiments. Comparisons between the exact and approximate mean-speed corrections are given in table 6. Agreement is reasonably good. In table 6 the exact mean speed averaged on the hull surface, $\bar{u}_0$, defined by (4.14), is not only non-zero but also dependent on Froude number. The free-surface effect on the velocity profile on the body surface would, significantly, depend upon whether the ship model would be towed in a shallower towing tank or in unbounded water. The present study indicates that the approximate speed-correction formula satisfactorily treats the seemingly complicated free-surface effect on the mean-speed correction on the body.

Wind tunnel experiment

As a second example, the blockage effect was considered for a wind tunnel having a uniform circular cross-section of R_0. The specific body geometry considered was a prolate spheroid with its meridian profile given by

$$\frac{x^2}{a^2}+\frac{R^2}{b^2} = 1 \tag{3.17}$$

for the special case when $a/b = 4$.

The potential flow for the axisymmetric boundary configurations considered herein could have been computed by the conventional method of integral equations; i.e. the axial source and doublet distributions or the vortex sheet on the surface, etc. as discussed in Landweber (1974). However, the velocity potential has been computed by the finite element method. Computations have been made for $R_0/b = 1{\cdot}25$, $1{\cdot}5$, 2, 3, 4, 5, and 15 for $a/b = 4$. When $R_0/b = 15$ was computed, the effect of the tunnel wall on the body surface was as negligibly small as if the body were moving in an infinite fluid. The value of $\bar{u}_0$, defined in (3.15), computed by using the result of $R_0/b = 15$, was 0·08185, whereas that computed by using the exact analytical result for the unbounded water, i.e. $R_0/b = \infty$, given in Lamb (1932) was 0·08156.

The computed velocity potential ϕ is shown in figure 3 for $R_0/b = 1{\cdot}25$, $1{\cdot}5$, and 15. To illuminate the assumption made to obtain the present approximate mean-speed correction, figure 3 shows straight lines drawn from the origin to the asymptotic values of $\frac{1}{2}K$ at the downstream stagnation point $x = \frac{1}{2}L$.† The slope of each straight line is equal to the speed correction defined by equation (3.4). Owing to the skew symmetry of ϕ with respect to $x = 0$, the result for the upstream half-body can be obtained from the downstream potential shown in figure 3. The normalized perturbation velocity potential increases monotonically from a value slightly lower than $-\frac{1}{2}K$ at the upstream stagnation point to slightly higher than $\frac{1}{2}K$ at the downstream stagnation point on the body surface. However, the potentials at $R = 1{\cdot}25b$ approach monotonically almost the asymptotic values at both ends for $R_0/b = 1{\cdot}25$ and $R_0/b = 1{\cdot}5$.

In table 7, the approximate mean-speed correction for three-dimensions given by 3 (a) in table 4 is compared with the exact mean-speed correction computed from (3.15). Also shown in table 7 are the approximate speed corrections given by (3.10) and by the Lock and Johansen formula which is given in Pope (1947) as

$$\Delta\bar{u} = 2{\cdot}391\left(\frac{b}{R_0}\right)^3. \tag{3.18}$$

† In this problem a slightly different form of the infinity conditions, (2.6) and (2.7), is used for simplicity: $\lim_{x\to-\infty} \phi = -\frac{1}{2}K$; $\lim_{x\to\infty} \phi = \frac{1}{2}K$.

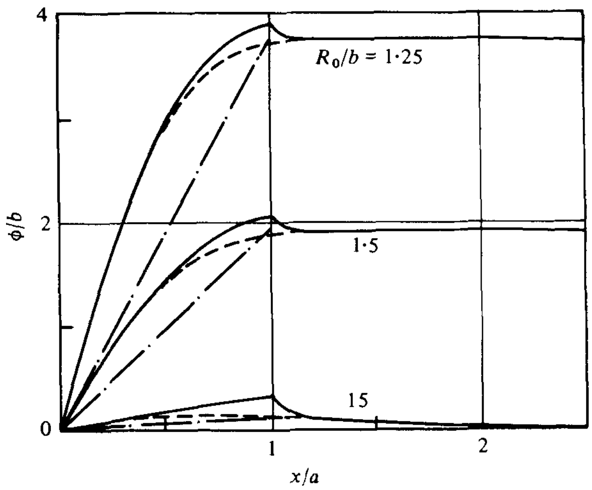

FIGURE 3. Velocity potential for a spheroid ($a/b = 4$) in a wind tunnel with a circular cross section of radius R_0. ——, on body; ----, at $R/b = 1{\cdot}25$; –·–, linear potential variation assumed in the present speed correction formula (the slope is the speed correction).

		$\Delta\bar{u}$			$\frac{2}{3}\left(\frac{R_0}{b}\right)^{-2}$
R_0/b	$\bar{u}$	Exact numerical result	Present formula 3 (a) in table 4	Lock and Johansen [equation (3.18)]	[from equation (3.10)]
1·25	0·98204	0·90050	0·93965	1·22419	0·42667
1·5	0·52285	0·44129	0·47716	0·70844	0·29630
2	0·26702	0·18546	0·21559	0·29888	0·16667
3	0·14442	0·06287	0·08505	0·08856	0·07407
4	0·11088	0·02932	0·04624	0·03736	0·04167
5	0·09748	0·01593	0·02920	0·01913	0·02667
15	0·08185	0·00030	0·00320	0·00071	0·00296

TABLE 7. Comparisons of mean-speed increments on a spheroid in a wind tunnel computed by the present approximate speed correction formula and by a numerical method. ($\bar{u}_0 = 0{\cdot}081557$ obtained by Lamb was used.)

In table 7, the comparisons show that results obtained by (3.10) and (3.18) give too low and too high values, respectively, while the present formula provides a better approximations when $R_0/b < 2$.

In figure 4, computed values of the added mass coefficient and the mean-speed correction $\Delta\bar{u}$ are shown as a function R_0/b. It should be noted in figure 4 that for $b/R_0 > 0{\cdot}765$, the contribution of the added mass to the previously described mean-speed correction from table 4 is more dominant than that of the displaced volume, i.e. $m' \equiv m/\rho V > 1$. This finding indicates that a crude correction formula given in (3.10), based on only the local cross sectional area of the body using one-dimensional theory, may not always provide a good approximation of the mean-speed correction when the added mass coefficient is not small, i.e. when $m' = O(1)$.

Comparisons between the present formula and the exact numerical mean-speed

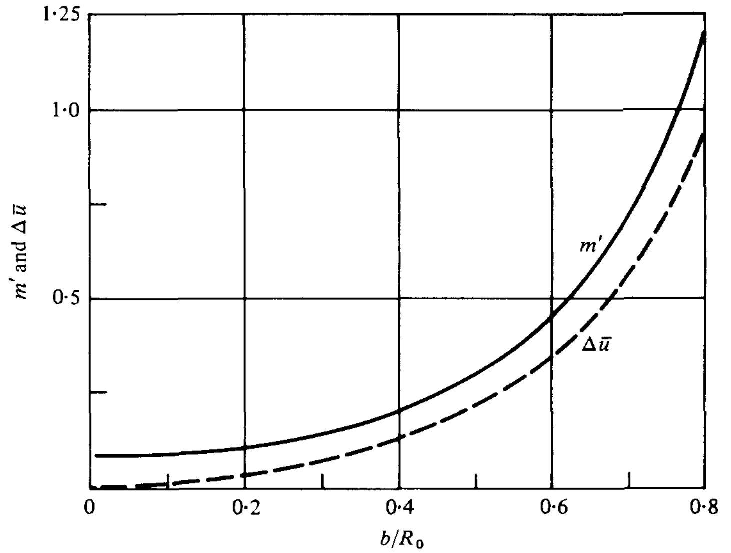

FIGURE 4. Added mass coefficient m' and speed correction $\Delta\bar{u}$ for a spheroid in a circular wind tunnel.

correction are favourable. More details about applications can be found in Bai (1978*b*). Further investigation is necessary to take into account other blockage corrections due to viscous effects such as flow separation and wake displacement thickness effects.

Sinkage

From the mean-speed increment given in (3.5), we can obtain an approximate expression for the additional sinkage η due to blockage by using the linearized Bernoulli equation as

$$\eta = \frac{U^2}{g}\frac{V(1+m')}{LA(1-F_H^2)}. \tag{3.19}$$

The sinkage can also be expressed in non-dimensional form by

$$\frac{\eta}{L} = \frac{F_H^2}{1-F_H^2}\frac{HV}{L^2A}(1+m') \tag{3.20a}$$

or

$$\frac{\eta}{L} = \frac{F_H^2}{1-F_H^2}\frac{V}{L^2W}(1+m'). \tag{3.20b}$$

One should note in the previous formula that if the tank width W increase indefinitely, while the water depth H is kept finite, the additional sinkage due to any finite-bottom effect is zero compared to the unbounded water. However, the change in sinkage due to the finite (or shallow) bottom, compared with sinkage in unbounded water ($W = \infty, H = \infty$) is significant. This obvious discrepancy in the present formula is because the directly related to the vertical component of the hull pressure force, thus the contribution to the sinkage is much more significant from the bottom than the sides of the ship. In other words, the vertical component of the normal vector on the ship hull surface plays the role of a weighting function in computing the sinkage correction; whereas, a mean-speed correction on the entire ship surface is used when computing the blockage correction to the frictional drag. Therefore, the present sinkage correction formula has a limited validity, i.e. only when the local speed increment along the girth line around each cross section of the ship is nearly uniform. This is physically the case when the tank width is approximately the same as the water depth. It should be noted that to obtain the sinkage correction, for example when the water depth is small while the width is infinite, the present formula should not be used. In this case, more detailed information concerning the local speed correction is necessary to compute the sinkage correction due to shallow water.

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Appendix A

In a simple analysis, using the eigenfunction expansions, it is shown that major portion of the potential jump does occur along the length of a slender body. For simplicity, a potential flow in an axisymmetric wind tunnel is considered. The wind tunnel has a uniform circular cross of radius R_0, and the testing body is also assumed to be axisymmetric with the body length L and the maximum radius b. Further, we assume that the testing body is long and slender and the tunnel radius is comparable to the maximum body radius b, that is, $L/R_0 \gg 1$. It should be noted that if $L/R_0 < 1$, then the blockage effect would be negligibly small; therefore, we rule out this case.

First, let us introduce a fictitious plane surface vertical to the x axis at the location barely touching the rear stagnation on the body. Otherwise, the testing body does not intersect with this fictitious plane. Here the uniform stream is from $x = -\infty$ to $x = \infty$, letting the intersection point of the x-axis and the fictitious plane, i.e. disk, be the origin, i.e. $x = 0$ and $R = 0$. The perturbation potential has to satisfy Laplace's equation in the fluid and homogeneous Neumann condition on the tunnel wall $R = R_0$, and the potential approaches to a constant C_0 (which will be determined later) as $x \to \infty$. The proper junction condition has to be satisfied on the fictitious plane (i.e. $x = 0$) later. We can express the general solution of the perturbation potential ϕ in a half-infinite subdomain $R \leqslant R_0$ and $x \geqslant 0$ as

$$\phi(x, R) = \sum_{i=0}^{\infty} C_i J_0(\lambda_i R) \exp(-\lambda_i x) \quad \text{in} \quad R \leqslant R_0 \quad \text{and} \quad x \geqslant 0, \tag{A 1}$$

where the infinite discrete eigenvalues λ_i, $i = 0, 1, 2, \ldots$, are defined by

$$J_1(\lambda_i R_0) = 0 \tag{A 2}$$

and C_i is the coefficient to be determined by imposing a proper juncture condition on the plane $x = 0$. Here J_0 and J_1 are the Bessel function of the first kind of order 0 and 1, respectively. The first few eigenvalues λ_i in equation (A 2) are given as

$$\left.\begin{aligned} \lambda_0 R_0 &= 0, \\ \lambda_1 R_0 &= 3{\cdot}832, \\ \lambda_2 R_0 &= 7{\cdot}016. \end{aligned}\right\} \tag{A 3}$$

To show analytically that the major portion of the potential jump occurs along the length of the body, it suffices to show that the value of the potential, approaches exponentially fast to a constant C_0 along the x axis ($x > 0$), then it suffices to examine only the slowest decaying term in equation (A 1). Under the assumption made earlier, we can examine a specific case, for example, if $L/R_0 = 30$, then the slowest decaying term, i.e. $i = 1$, in equation (A 1), becomes

$$C_1 J_0(\lambda_1 R) \exp\left(-3{\cdot}832 \frac{L}{R_0} \frac{x}{L}\right) = C_1 J_0(\lambda_1 R) \exp\left(-114{\cdot}96x/L\right). \qquad \text{(A 4)}$$

In this example, it can be seen that at $x = 0{\cdot}03\, L$ the slowest decaying term already decays to 3 per cent of the value $C_1 J_0(\lambda_1 R)$ at $x = 0$. It should also be noted that the potential approaches to a constant, C_0, much faster since the rest of the terms ($i > 1$) decay much faster than the second term ($i = 1$) in equation (A 1). One may obtain the same result near the front stagnation point simply by reversing the flow in the previously results.

However, the present simple analysis fails, if L/R_0 is not very large. When L/R_0 is not very large, the Green's function would seem to be another alternative. However, by examining a simple point source and a sink with the same strength located along the centre-line $R = 0$ in a circular tube given in Landweber (1974), it is difficult to obtain the previous results analytically unless a numerical computation is involved for computing integral expression for a point source given in terms of the modified Bessel function of the second kind.

For a more general case of free-surface flows in a canal, one may examine exponentially-decaying behaviour in the local disturbance both upstream and downstream, in a similar manner.

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